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R E P O R T R E S U M E S

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THE USE OF MATHEMATICAL PROGRAMING TO SOLVE CERTAIN PROBLEMS IN PUBLIC SCHOOL TRANSPORTATION.

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A PROGRAM THAT WOULD DEVISE OPTIMAL ROUTES FOR SCHOOL BUSES WAS DEVELOPED. ONE OF THE TWO MODELS DEVELOPED FOR THIS STUDY, CALLED THE HEURISTIC MODEL, WAS A HAND PROCEDURE BASED ON THE ASSUMPTION THAT, IN AN EFFICIENT SCHOOL BUS SYSTEM, STUDENT MILES SHOULD BE MINIMIZED RATHER THAN BUS MILES. THE OTHER MODEL WAS DESIGNED TO BE USED WITH HIGH SPEED COMPUTERS AND IS CALLED THE COMBINATORIAL ALGORITHM. GIVEN A MAP OF A TYPICAL SCHOOL DISTRICT, SIX EXPERIENCED SCHOOL OFFICERS WERE ASKED TO DESIGN SCHOOL BUS ROUTES ACCORDING TO THEIR USUAL PRACTICE. ALSO, A CLASS OF GRADUATE STUDENTS DESIGNED ROUTES FOR A MINIATURE SCHOOL DISTRICT. THE HEURISTIC MODEL YIELDED ROUTES WHOSE BUS MILES WERE COMPARABLE WITH THOSE ROUTES DESIGNED BY EXPERIENCED DESIGNERS. A NOTICEABLY SMALLER NUMBER OF STUDENT MILES WAS OBTAINED BY THE USE OF THE HEURISTIC MODEL. THEREFORE, IT MAY OFFER A RAPID MEANS OF OBTAINING A REASONABLY GOOD ROUTING SYSTEM. THE COMBINATORIAL ALGORITHM WAS SHOWN TO PROVIDE A BETTER SOLUTION THAN EITHER THE HEURISTIC MODEL OR THE GRADUATE STUDENT DESIGN. THE PROGRAM SUGGESTED IN THIS REPORT FIRST GENERATES SCHOOL BUS ROUTES BY CONSIDERING ONLY THE NUMBER OF CHILDREN (LOAD) AND THEN COMPUTING BUS AND STUDENT MILES FOR EVALUATIVE PURPOSES. IT WAS RECOMMENDED THAT AN ATTEMPT BE MADE TO DETERMINE THE FEASIBILITY OF A PROGRAM THAT WOULD CONSIDER NUMBER OF STUDENTS AND DISTANCES CONCURRENTLY. (JL)

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The Use of Mathematical Programming to Solve Certain Problems in Public School Transportation

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Project Director

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1961

Preface

The research reported herein was undertaken in accordance with a contract with the U. S. Office of Education as outlined in the appendix to the contract (Cooperative Research Project No. SAE-8576). This research was intended to investigate mathematical programming of transportation problems of public schools. As the project developed, it became evident that this problem was more closely related with the area of operational research than mathematical programming.

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Chapter 1. Introduction

The process of formulating a problem and then constructing a model to represent the observed phenomenon has been carried out systematically in the physical sciences. This type of deductive process is most frequently encountered under the general heading of "theoretical physics." In other fields of study the use of deductive methods remains a relative novelty. However, there is an increased number of scholars who are attempting to apply an axiomatic method in model construction. One recent attempt in education has been described by Mort and Furno in "Theory and Synthesis of a Sequential Simplex, a Model for Assessing the Effectiveness of Administrative Policies" (1960). The present study is concerned with a model for designing school bus routes.

Designing bus routes tends to be an intermittent, tedious task and one for which little formal training has been provided. Murray (1961) reviewed college catalogues of 16 states (Alabama through Kansas alphabetically) and found two universities and one teachers college offering courses in school transportation. Only in the catalogue of the Alabama Polytechnic Institute did the course description mention the routing of buses. If one considers that the cost of school transportation is over $440,000,000 (Murray, 1961) annually, it would seem logical that this vast expenditure would attract attention and study. A review of the studies made in the field of school transportation suggests that research has been limited to the survey-questionnaire type of reporting. No manipulation of variables was found.

Chapter 2. Review of Related Research

In order to become acquainted with variables that have been considered in designing bus routes, the researchers made a comprehensive review of the research published between 1930 and 1960. Also reviewed were articles in the area of certain mathematical procedures applicable to the study.

Public School Transportation

The review of public school transportation studies included those cited or appearing in the Encyclopedia of Educational Research (1940, 1950, 1960) U. S. Office of Education publications, professional journals, handbooks, bulletins, and other similar publications.

These sources revealed that many factors in school transportation have been studied and that most of the work described has been limited to outlining standards for school buses, standardizing safety devices, perfecting means of allocating state funds for transportation services and making surveys. The articles cited below were arbitrarily selected and are believed to be representative of the research in the respective areas.

Standards for school buses have been reported by Hutchins (1938), Butterworth (1941), Belknap (1950), Culp (1955), National Chief State School Officers (1956), and the state departments (1949, Kansas 1959). The area of bus safety devices has been discussed by the N.E.A. (1936), Belknap (1950), Mattern (1958), Peters (1958) and Zaun (1958). Methods and procedures of allocating funds on a state basis have been reported by Burns (1927), Johns (1928), Reusser (1932), Hutchins (1941), and McLure (1947), while Morphet (1932, 1939), Roberts (1936),

Lambert (1937), Meadows (1940), Stapley (1947), Reusser and Waterman (1949), and Belknap (1954) have made surveys of transportation services.

Few articles were found regarding the actual routing of buses. These articles ranged from mere descriptions of routes to explanations of more complex procedures of routing buses by school personnel. An early article by Walker (1930) listed vague and somewhat philosophical suggestions. Typical of this approach was the statement that bus routes should be laid out to insure the best possible service to each student.

Reeder (1939) classified bus routes according to three types: "circular," "shoe string," and "spur." He added that no general statement could be made regarding economy in miles that might result from the use of either type route. Juckett (1943), a Michigan State University bulletin (1956), and a Florida Public School Transportation handbook (1959) suggested situations in which these three types of routes might be used.

Monetary economy as a consideration in bus routing was treated by Linn (1956) and Kirkpatrick (1960). Both advocated, without supporting evidence, the use of larger buses as an economy measure. Featherston (1960) stated that the increased sale of larger buses reflected the special needs of local transportation systems.

Several writers have discussed the responsibilities of school personnel pertaining to the routing of buses. Noble (1940) stated that the matter of laying out bus routes was "largely a matter of local initiative." He recommended that schools keep an accurate, up-to-date transportation map and that state departments provide more aid in routing buses.

Robinson (1947) asserted that the first step in laying out bus routes was a pupil population survey followed by a very definite school board transportation policy. Writing in a more explicit manner, Butterworth (1952) pointed out that it was the principal's responsibility to plan routes that would best

serve the students. He suggested the system of making choices from alternative data that had been written down in parallel columns.

Belknap (1950, 1954), in suggesting how routes could best be laid out, recommended the use of maps with colored pins, conferences with bus drivers, description of roads, and the arrangement of time schedules by trial run. No procedure was given in either reference as to how routes should be laid out other than by "diligent study of local communities." Wimbish and Gilbert (1960), who cited the importance of maps and colored pins to indicate school locations, numbered thumb tacks and colored string to indicate different routes. Other factors mentioned were walking zones, loading and unloading buses, positioning bus stops, and the differences in route planning for rural and urban areas.

Representative of the type of guidance available to people designing school bus routes is that given by Stapley (1947):

> Those in charge of planning bus routes should think first of the optimum pupil load and bus route length in order to visualize the type route which should be created. They should then begin experimentation in the creation of routes that will fit into a logical design. The sum total of all bus route mileage should be the smallest that can be arranged. Efforts should be made to transport pupils on routes that do not cover unnecessary mileage. There is something wrong when planning results in two buses traveling the same road except when the road approaches a school. Since it is likely that many attempts will be made before the routes are finally established, a satisfactory beginning can be made by placing a sheet of tracing paper over a spot map. The solution for any situation will be reached through perseverance and ingenuity, for which there is no short method.[1]

Mathematical Techniques

A review was made of the mathematical literature applicable to school bus routing.

The only writer found in educational literature who perceived bus routing with mathematical implications was Morphet (1939). He defined bus routing

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[1]Maurice E. Stapley, "Suggested Procedures for Securing Economic and Efficient Pupil Transportation," Bulletin of the School of Education of Indiana University, XXIII, No. 4, Bloomington, 1947. p. 10.

as the process of finding the fewest possible lines not to exceed a maximum length that would fit a given situation, taking into consideration all the factors involved. He added that this process was no easy task because it could not be solved like a mathematical equation.

In 1937 when Flood (1956) was struggling with problems in a new Jersey school bus routing study, A. W. Tucker called to his attention the related work of Ball and Hamilton which was concerned with finding the number of different routes possible over a specified configuration. (The Hamiltonian Game consisted of the determination of a route which would pass only once through each of the angular points of a regular dodecahedron.) Since Flood was not interested in optimal routing, no effort was made to investigate the application of this mathematical technique.

More recently, Henderson and Schlaifer (1954) made a summary of the research and practical applications of mathematical techniques in traffic management transportation and in the industrial areas of tool design and utilization, production management and control, and in marketing.

A comprehensive review was made of the traveling salesman problem and its solution by Barachet (1957), Dantzig (1952, 1957, 1959), Heller (1954, 1955), and Riley (1959).

Other articles reviewed included the optimum routing of gasoline delivery trucks by Dantzig (1959), and highway trucks by Caldwell (1961), the routing of telephone calls by Wilkinson (1956), and other technical articles by Pilliod (1952), and Lee (1955). With the exception of the studies pertaining to telephone dial systems, most of these articles tended to be academic in nature. On many occasions the inference was given that no practical application of a particular method had been made as yet.

Early in this study the traditional traveling salesman problem seemed to be closely related to the school bus problem, i.e. finding the shortest route

that visits each point on a given map. In school situations, however, all points are not visited by one, and only one, bus. Some points may be visited by two or more buses; other points by one, and only one, bus; and no bus has the capacity to visit all points. Also the greatest number of points visited as cited in the usual traveling salesman problem is less than 50, whereas it is not uncommon to have 200 pick up points in a school district. In addition present mathematical models associate a value of one to each point, whereas in the present study there may be one, two, or twenty children at a point. What may be needed, therefore, is a mathematical procedure that will minimize distances between points in an array and simultaneously will accumulate the weighted values for each point under restraints on size of each accumulation. The sizes of the accumulations also vary according to the different sizes of school buses.

Summary of Related Research

There is evidence that the designing of school bus routes is being done by relatively untrained personnel and at such intervals that the practice effect may be of little importance. Judging from the literature, one may believe that most school bus routes are dependent solely upon size of buses available and the lowest cost of operation. Other factors tend to be secondary. Thus, it is unlikely that the "best route systems" have been arrived at by a majority of the people designing school bus routes. Moreover, the best route system by any criterion is actually unknown. It is impractical to enumerate all possible routes for all possible sizes of school buses and to observe maxima and minima in regard to student miles and bus miles. It would seem, therefore, desirable to develop a procedure for finding "optimal school bus routes" applicable in any given situation and through its use to minimize the element of trial and error inherent in present day practices.

Two approaches to a solution of this problem were attempted, the combinatorial and the heuristic.

Chapter 3. Method

Initially it was planned to compare the route systems of six school districts with those suggested by a deductive procedure. However, while raw data were being collected, it was found that certain inconsistencies existed between the number of students being regularly transported and the number of students that were indicated on the schools' transportation maps. It was also discovered that the number of students being assigned to a particular bus was greater than the capacity of the bus. In one instance, 93 children were assigned to a bus with a capacity of 54 seats. Because of these occurrences, the experimental design was changed to a categorical approach.

The plan adopted in this study was to secure transportation maps of six representative schools on which were given the locations of the pupils, schools, and roads. The maps were then given to school officials who were experienced in planning school bus routes. Each school officer designed a school bus system for a particular school district. Meanwhile, using a model developed in this study, school bus routes were designed for each school district. The routes designed by the experienced men and those designed by means of a model were then compared and evaluated.

Selection of Route Designers

The six experienced school officials who independently constructed the bus routes for the six school districts were selected on the basis of personal knowledge of their professional work and upon recommendations of leaders in the field of school administration. All of these men were then engaged in

transportation administration and the designing of bus routes for their respective school districts. Each official was given a modest honorarium. Hereafter, these school officials will be referred to as route designers.

Collection of Data

The school districts in Mississippi were examined in regard to location, topography, road patterns, geographical barriers and transportation records. Three types of areas were selected by the project director as representing the range of transportation problems encountered in most school districts. Two of these districts had geographical barriers (Mississippi River, Gulf of Mexico), two districts had rectangular road networks, and two districts had roads that tended to follow topographical ridge lines. These school districts will be identified as A, B, C, D, E, and F. Detailed information may be found in Appendix A.

Visits were made to the schools and to the state department of education to secure transportation records. Although the State of Mississippi required school districts to file a transportation plan, entitlement roster, driver, and school attendance records, the only records deemed pertinent to this study were the transportation maps. The transportation maps contained symbols denoting route numbers, bus stops, and the number of children located at each stop (See Appendix A.) These data from the maps were standardized in the following manner:

1. At boarding points or pickup points the number of students was recorded in arabic numbers inside small circles.

2. The location of the school was indicated on the map by a black rectangle.

3. Whenever possible the students were grouped at the centroidal point on each road segment.

In order to simplify the procedure, one attendance center was used for each school district.

The route designers were given detailed written instructions and related materials, consisting of a letter of transmittal, a sample map, a one inch to one mile scaled highway department map, tables to record preparation time and route information, and crayons. (See Appendix B.)

The specific instructions given to the route designers were:

1. Design the most efficient possible school bus routing system in the same manner you normally employ.

2. Only school buses of 48, 54, 60 and 66 student capacity could be used. Any combination of these sizes was permissible.

3. The number of students assigned each bus could not exceed the capacity of the bus.

4. All types of roads were to be considered as equally good.

5. No child was to walk to a remote pickup point.

Upon receipt of the data from the route designers and completion of the routes by using a model, the length of each bus route was computed in terms of bus miles and student miles.

Graduate Student Designers

In a graduate class in educational administration the evaluative criteria by which a school's transportation system might be judged was explained and discussed in detail. (These criteria will be listed below.) Emphasis was placed upon the concept of bus and student miles. These students were then asked to devise bus routes that would result in the fewest number of student miles for a miniature school system. It is to be noted that the instructions were slightly different from those given to the six experienced route designers; namely, the latter were told to devise the most efficient routing system. The

results of the routes prepared by the graduate students were compared with the routes suggested by the two models.

Chapter 4. Evaluative Criteria for Transportation Systems

Selection of the criteria by which the effectiveness of a transportation system could be evaluated proved to be one of the major problems encountered in this study. A period of approximately four months was devoted to attempts in designing school bus routes that would meet different criteria. More than 600 school bus routes were produced and evaluated. These criteria have been grouped below under the following categories: financial, safety, route network, bus size, miles traveled, and time.

Financial

Transportation systems may be evaluated by simply comparing the costs per student mile or costs per bus mile. It was found that both costs tended to be more dependent upon population density, area, and sizes of buses than on the efficiency of the routes. The costs per bus mile and student mile were found to be relatively low for a school district that was small with excellent roads and a large population. On the other hand, both costs would be far greater in rural consolidated school district that had a low density of population, poor roads and the necessity of transporting students over long distances. Therefore, cost per mile was not considered as a suitable criteria for evaluating a transportation system.

Safety

As was indicated in the review of research, the area of safety has been the subject of many articles pertaining to school transportation. Therefore, it seemed appropriate that school bus routes should be so designed to incorporate

safety oriented characteristics. The safety variables manipulated in the experimentation with different school bus routes were as follows:

1. Movement of school buses at road intersections.

At the usual road junction the bus may make a right hand turn, a left hand turn or be driven through the intersection. By counting the number of lanes of traffic involved in each maneuver, (for example, a right hand turn would involve driving in two lanes, and a left turn, four lanes) numerical values were assigned to each type of turn. Routes were drawn maximizing right hand turns and minimizing left hand turns.

Although it was anticipated that the resulting route configuration would be clock-wise circular networks, the actual routes did not appear to be different from any other routes. When a comparison was made with routes where no preference had been given to movement at intersections, very slight and inconsistent differences were observed between number of right and left hand turns. Therefore, the movement of buses at road junction was eliminated as a possible criteria of evaluating school bus systems.

2. Railroad crossings.

The school districts selected for this study had few railroad crossings and no route was designed in an attempt to minimize such crossings. Information from the National Safety Council (1960) concerning school bus accidents at rail crossings suggests that accidents are rare. Since there is no evidence that such accidents are rare because school administrators attempt to minimize railroad crossings for school buses, it may be spurious to evaluate routes according to the number of railroad crossings.

3. Backing the school bus.

Also included in the safety category was the assumption that backing a school bus increases the probability of an accident; therefore, such operation should be minimized. During the experimental period, routes were designed attempting

to avoid situations requiring a bus to be turned around. It was found that turnarounds were usually the resultant of spurs and the number could not be minimized by different routes.

4. The necessity of students crossing roads at pick-up points. It seems reasonable to assume that there is increased probability of having an accident when children have to cross a road in order to board a bus. This situation is contrasted with one where children do not have to cross a road to board a bus. Analysis made of the school population maps disclosed that children are rather equally distributed in regard to the side of the road on which they live. Therefore, it was concluded that it would be fruitless to attempt to design bus routes that would maximize children residing on the right hand side of the road.

Route Network

Many school bus routes were designed to meet the following criteria and their ramifications as related to road patterns and conditions:

1. Two or more buses should not be assigned to the same road unless the buses are filled and are on route to the school.

2. School buses should be assigned to a particular area. For example, if 54 children live in one well defined area, one bus with a capacity of 54 children would be assigned to transport these children.

3. An optimal school bus route does not cross itself.

4. School bus routes should be assigned to paved roads wherever possible.

5. School bus routes should be designed according to a consistent plan, such as circular routing.

6. The most efficient route to deliver children to their homes is the reverse of the most efficient route that delivers the children to the school.

Criterion 1 and 2 above are closely related. If two buses are assigned to pick up children on the same road, it follows that more than one bus has

been assigned to a particular area. It was found that efficient routes could be designed when two or more buses were assigned the same road. In some instances four buses could be efficiently assigned to one sector of the school district in which the total school population equaled the capacity of one bus. In such cases the buses would pick up children enroute to the school instead of picking up a load at a remote area and then driving directly to the school.

Criterion 3 was suggested by solutions developed for the traveling salesman problem; that is, the shortest optimal route joining all points in an array does not cross itself. The limitations of the traveling salesman routes were discussed in the review of related research. No experimentation was made on designing routes that did not cross themselves.

Criterion 4 above assumes that since paved roads permit faster service and reduce operating expenditures, school routes should be designed to make maximum use of such roads. Experience with designing routes indicated that in most situations paved roads and population density are related, thereby "paved roads" per se is a dependent variable and using it as a criterion is to be questioned.

Criterion 5 relates to designing a transportation system according to predetermined route networks, i.e., circular, string or spur routes. Several systems were designed in which routes were either all circular or all string routes. The circular routes tended to be longer than string routes and seemed to be more difficult to draw. Also, at the beginning of circular routes, the direction of the movement of the bus tends to be away from the school. Such a condition will increase the number of student miles because later the bus must reverse its direction.

Criterion 6 assumes that the most efficient route for the students to be distributed to their respective homes is simply the reverse of the route that is the most efficient in transporting the children to school. Failure was

experienced in attempting to design bus routes starting at the school and transporting the children home. Therefore this criterion was not evaluated in this study.

Bus Size

In the review of related research it was noted that the sizes of school buses continue to increase with each new model. Many school districts tend to maintain buses of the same capacity; that is, all buses have the capacity of 54 students. Whether such a policy of standardizing buses is a good one, from any standpoint, is really unknown. Consequently, many routes were drawn utilizing buses of the same capacity and buses with varying capacities. The analysis of these routes yielded many inconsistent results and no trends were noted. It was found that route systems having different size buses were more easily drawn.

Miles Traveled

The distances buses and students travel while enroute to and from school are easily computed and understood. Bus miles or speedometer readings are common criteria and of a type that needs no further explanation. The standard for a bus route in terms of student miles may be characterized by the accumulation of miles that each student rides. In other words, if six students rode five miles, four students rode three miles, and two students rode one mile, the accumulated student miles would be 44 and the bus miles, five.

Granted the assumption that schools are for the students and bus routes exist solely to transport these students, it follows that the pre-eminent criterion for evaluating bus routes would be in terms of students and not in terms of buses, costs, roads, or related factors. Therefore, student miles may meet the standard of evaluating school bus routes more readily than any previously discussed criteria.

Time

Closely related with student miles and also meeting the standards of evaluating routes is the accumulated time each student spends on the bus. Since existing maps are drawn in scale of miles and not time, time enroute was not used as a criterion for the purpose of this study. Although there is no evidence to support such a statement, it is believed that student miles and student time are closely related and that the optimal route for accumulated student miles would be the same route if accumulated student time had been used.

Chapter 5. Evaluative Criteria for Methods of Devising School Bus Routes

This section of the report will be devoted to the properties by which a school bus route model may be fairly judged. The order of these properties is arbitrary.

A bus route model should possess several properties.

1. It should be rigidly defined.
2. It should require no assumption on the part of the designer.
3. It should be readily comprehensible.
4. It should require elementary mathematical computations.
5. It should be affected as little as possible by different road patterns and geography.
6. It should be able to be terminated at any one point and started again without repeating any previous computation or review.
7. It should yield an optimal or near optimal solution.
8. It should be inexpensive.

These criteria will be used to evaluate both the combinatorial and heuristic model.

Chapter 6. Variables Used in Models

There were many variables that were studied for possible incorporation in the models, namely:

1. Bridges (weight capacity)
2. Size of buses
3. Type of road (federal, state, county)
4. Road condition (paved, gravel, dirt)
5. Gradient of road (steep, moderate, level)
6. Type of turn of intersections (right, left, other)
7. Turn arounds
8. One way streets
9. Railroad crossings
10. Miles traveled (bus, student)
11. Time spent in travel

Most of these variables were eliminated from the study because they were either too unwieldy or did not meet the criteria as outlined elsewhere. The variables actually used may be found on a typical school transportation map:

1. Road distance between pickup points.
2. Number of students at each pickup point.

Whenever possible, the students were grouped so that only one pickup point was established on each road segment.

3. Condition of road.

In the heuristic model no provision was made to consider road conditions per se. However, this would not exclude a route designer from considering this

factor because road conditions are shown on maps. In the combinatorial model, conditions of roads were incorporated by the use of multipliers whose value could be readily manipulated. For example, should the road between two points be paved, the road distance is multiplied by one; if the road is gravel, the distance multiplied by two; and if dirt, by three. These multipliers were arbitrarily selected; any other values may be used.

Chapter 7. Miniature School System

This section will describe a "miniature" school system that will be used to illustrate the ways the models devise school bus routes. The configuration of the miniature school system has only those features necessary to illustrate the respective models. Many other configurations could easily have been used and it is believed that the present one is not unique. If a study is made of the following descriptions and illustrative solutions of the miniature school system, the respective models will be readily understood.

The miniature school system consists of seven pickup points identified by the numbers 2, 3, 4, 5, 6, 7, and 8. The school is designated by the number 1. (In the section on the combinatorial solution of this school system, pickup point 4 will be partitioned and thus nine pickup points will be considered). The total number of children to be transported is 45 and the number of children at each pickup point is as follows:

Pickup point	Children
2	6
3	4
4	12
5	10
6	9
7	3
8	1

The distance between the respective points is given in Figure 1.

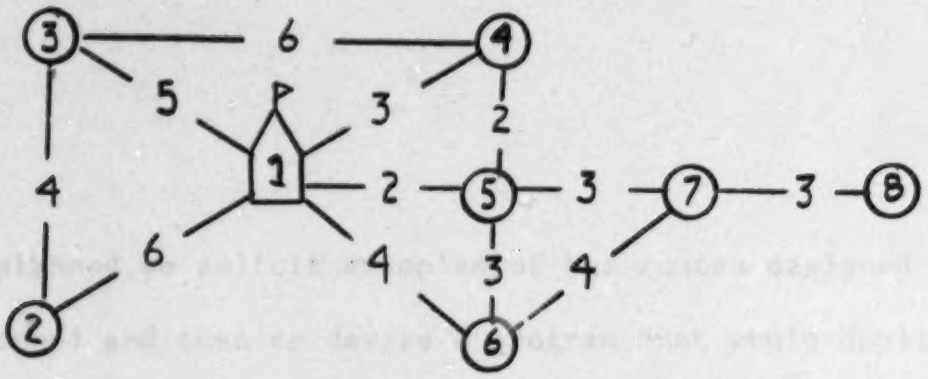


Figure 1. Skematic Diagram of Miniature School System

The school buses for this miniature system are of three types: buses having a load capacity of 12 children, buses with 15 children capacity and those buses with the capacity of 18 children. A restraint is imposed upon the system in which a maximum of three buses may be used. There are six possible combinations of the three types of buses that could transport the student population of 45 children:

Bus 1:	12	12	15	15	15	18
Bus 2:	18	15	15	15	18	18
Bus 3:	18	18	15	18	18	18
total	48	45	45	48	51	54

As in all solutions presented in this report, no "over-loading" of a school bus is permitted. Also, all distances will be reported as "miles." Perhaps "units" such as "bus units" and "student units" would more accurately describe distances but it was believed that the term "miles" would be more readily understood.

Chapter 8. Heuristic Model

Originally it was planned to solicit examples of bus routes designed by experienced school personnel and then to devise a program that would duplicate the work accomplished by these men. In turn a question arose concerning the efficiency of these routes designed by experienced school administrators, or stated in another manner, is there a more efficient route system than one drawn by trained personnel and if so, what is it? The search for a better school bus system was culminated in the development of a hand-procedure of drawing school bus routes. Because of the inherent limitations of this procedure, it will be referred to as the "heuristic model."

Description of the Heuristic Model

The heuristic model will be described first by giving a brief description of the major sequential steps in devising a routing system, second, by giving all the steps and related information, and third, by illustrating its application to the miniature school system problem.

The heuristic model consists of a set of sequential operations. Starting a bus at the farthest pickup point from school, the shortest route to the school is drawn. Atthis time the map is inspected for any children who live adjacent to this route who would not be likely to be included on any subsequent route. These latter children are called isolates. The number of children on the shortest route and the number of isolates are added and the sum is called load. If the load is equal to the capacity of a bus, the route is established. If the load is greater than the capacity of a bus, those children residing near the school are removed from the load until capacity of the bus is reached.

Should the load be less than that of a bus, the shortest route is lengthened by the smallest possible distance to include additional children until load capacity is reached. The pattern of the final route may be altogether different from the original "shortest route." The only points to be included on all trial routes will be the farthest point, the isolates created by each trial route, and the school.

A complete sequential description of the heuristic model is as follows.

Heuristic Model

Data to be given:

1. Map giving location of students (pickup points) and school.
2. Total number of students to be transported.
3. Minimum number of buses and capacity of each bus required to transport student population.
4. Maximum number of buses and capacity of each bus required to transport student population.
5.

Bus number (Identification number of bus)	Capacity (48, 54, 60 or 66)
#1	___
#2	___
#3	___
---	---

Sequential Steps:

1. Is there a bus that has not been assigned to a route?
   a. If answer is "yes", assign bus to a route.
   b. If answer is "no", go to 2.
2. Is there a point that has not been assigned to a route?
   a. If answer is "yes", go to 3.
   b. If answer is "no", school bus problem has been solved.
3. What point not already assigned is greatest distance from school? (Starting point)
4. What is the shortest route from starting point to school? (Basic route)

5. Will this basic route create an isolate?
   a. If answer is "yes":
      (1) What is shortest route from starting point to school that will include isolate?
      (2) Will this route create an isolate?
         (a) If answer is "yes", return to (1) and adjust accordingly until a route is designed that includes: starting point, isolates, school, and is the shortest route that will join all of these points.
   b. If answer is "no", go to 6.

6. What is the number of children on this route? (Load)

7. Is the load of this route equal to or greater than the minimum bus load and equal to or less than maximum bus load?
   a. If answer "yes", Route No. 1 and Bus No. 1 are fixed. Go to 8.
   b. If answer is "no", scan map, increase route the smallest possible amount that will include at least one additional pick-up point. The only pick-up points that must be included on this route are the starting points, isolates and the school. As each new route is designed, go to (5) and repeat process until answer to (7) is "yes".

8. Arrange route to yield minimum number of student miles. (SM)

9. Arrange route to yield minimum number of bus miles. (BM)

10. Add student miles on previous routes and student miles on present route to give current adjusted student miles to date.

11. Add bus miles on previous routes and bus miles on present route to give current adjusted bus miles to date.

12. Compute number and sizes of subsequent buses.

13. Go to 1.

Illustrations of the Heuristic Model

Below are given several solutions to the miniature school system problem. The first solution might be called an unrestraint solution, that is, any convenient combination of buses would be acceptable. In this case, the three buses will be of 15, 18, and 18 capacity. The actual sizes of the buses would be determined concurrently with solving the routing system.

First heuristic solution to miniature school system:

Given: 1. Number of students: 45

2. Seven pickup points: 2, 3, 4, 5, 6, 7, 8

3. School: 1

4. Minimum number of buses: 3
   Maximum number of buses: 3

   See disscussion under Miniature School System for possible combinations of school buses to transport the student population.

5. No restraint is imposed upon the combination of buses that will be used.

Successive steps in solution

Bus Route No. 1

1.	What point not already assigned is the greatest distance from school?	#8
2.	What is shortest route from that point to school?	#8-7-5-1 (8 miles)
3.	Will this basic route create an isolate?	Yes, point #6 will be isolate.
4.	What is the shortest route from starting point to school that will include the isolate?	#8-7-6-1 (11 miles)
5.	Will this route create an isolate?	No
6.	What is the number of children on this route?	13
7.	Is this value greater than capacity of smallest bus (12) and less than largest bus (18)?	Yes
8.	Route No. 1 is determined.	#8-7-6-1
9.	Arrange route to yield minimum student miles. Arrange route to yield minimum bus miles.	#8-7-6-1 #8-7-6-1
10.	Student miles to date.	71
11.	Bus miles to date.	11
12.	Compute sizes of subsequent buses.	45 - 13 = 32 If the two subsequent buses are to transport 32 children and no bus may have a capacity over 18, the smallest load any bus can have is 14 and the largest load of 18 students.

Bus Route No. 2

1.	What point not already assigned is the greatest distance from school?	#2

2.	What is the shortest route to school?	2-1 (6 miles)
3.	Will this route create an isolate?	No
4.	What is the number of children on this route?	6
5.	Is this value greater than smallest computed capacity?	No
6.	Route must be increased and must include both B and School. The shortest route would be:	2-1-5-1 (8 miles)
7.	Are any isolates created by route 2-1-5-1?	No
8.	What is the number of children on this route?	16
9.	Is it greater than 14 and less than 18?	Yes
10.	Route No. 2 is established.	
11.	Minimize route for student miles:	80 miles
12.	Minimize route for bus miles:	10 miles
13.	Student miles to date.	151 miles
14.	Bus miles to date.	21 miles

15. Computation of size of subsequent bus.

45 - 13 (Route No. 1) - 16 (Route No. 2) = 16 for Route No. 3.

Bus Route No. 3

1.	Sum of all students who have not been picked up:	16
2.	Is this value greater than minimum capacity and less than maximum capacity of remaining bus?	Yes
3.	Route No. 3 is established.	
4.	Minimize route for student miles (3-4-1)	72 miles
5.	Minimize route for bus miles (3-4-1)	9 miles
6.	Student miles to date.	223 miles
7.	Bus miles to date.	30 miles

Second heuristic solution to miniature school problem:

In a situation where greater restraints are imposed on the solutions, such as, each of the three school buses will have the capacity of 15 and

only 15 students, the condensed heuristic solution is as follows:

Bus No. 1

8	Point greatest distance (8 miles)
8-7-5-1	Shortest route (8 miles) 6 becomes an isolate
8-7-6-1	Next shortest route that includes isolate (11 miles) Load is less than 15
8-7-6-5-1	Next shortest route (12 miles) Load exceeds 15. Drop off children at 5 until 15 is reached. Adjust for minimum distances.

Route No. 1 becomes 8-7-6-5 (2 students picked up) -1

Bus No. 2

2-1	Point greatest distance (6 miles) Shortest route (6 miles) Load of 6 students is less than 15
2-3-1	Next shortest route (9 miles) Load of 10 students is less than 15
2-1-5-1	Next shortest route (10 miles) Load 14 is less than 15.
2-1-4-1	Next shortest route (12 miles) Load of 18 students is greater than 15 Drop off students at D until 15 is reached Adjust for minimum distances

Route No. 2 becomes 2-1-4 (9 students picked up) -1

Bus No. 3

3	Point grestest distance (5 miles)
3-4-5-1	All points not picked up to date Load of 15 is correct Adjust for minimum distances

Route No. 3 becomes 3-4-5-1 (10 miles)

Total student miles 255

Total bus miles 34

The latter problem (three buses with 15 and only 15 students per bus) was administered to 24 graduate students in educational administration courses.

The lowest number of student miles as computed by any graduate student was 261 as compared with 255 student miles when routes were devised by the heuristic model. (It is to be noted that the heuristic method yields a total of 219 student miles when students on Route 2 are left at the school the first time the bus comes to the school.) The minimum number of bus miles to transport the students in the miniature system appears to be 29 miles, as compared with 34 given by the heuristic model.

A unique characteristic of the hand solution is that if one uses three buses of 12, 15, 18 student capacity, and changes the order of assigning buses, i.e. 12, 15 and 18; 12, 18 and 15; 15, 12 and 18; 18, 12 and 15; 18, 15 and 12, a range of bus miles and student miles are obtained. For example, by using the heuristic model and assigning students to the buses in the following order: 12, 15 and 18 a total of 275 student miles and 35 bus miles are obtained. On the other hand, when using the order of 18, 15 and 12, 255 student miles and 34 bus miles result.

In experimenting with the heuristic model in the miniature system, certain trends were noted which were also observed when the model was used in other school systems. The model will devise routes where the number of bus miles is slightly greater and student miles are noticeably shorter than other methods used to date. These data suggest that routes having optimal bus miles are not congruent with routes having optimal student miles.

Application of Heuristic Model to Six School Systems

Comparisons of the school bus routes that were prepared by the six experienced route designers and the routes resulting from the application of the heuristic model are given in Table 1. As had been observed elsewhere, the total number of miles the buses traveled in routes designed by experienced personnel was less than the distances given by use of the heuristic alogrithm.

Table 1. Comparison of School Bus Routes Designed by Six Experienced Officials and Routes Designed by Using a Heuristic Model.

School System	Number of students transported	Experienced Officials		Heuristic Model	
		Student miles*	Bus miles*	Student miles*	Bus miles*
A	406	4161	139	4023	148
B	623	7671	244	7273	243
C	326	4089	135	3421	142
D	227	10348	223	9099	227
E	928	7065	173	6084	195
F	666	9514	278	8126	282

*It is to be recalled that "miles" are actually units of distance. In most cases by multiplying the miles given in the table by 2/3, actual miles can be obtained. Differences are due to the ratio used to reduce maps to 8 x 11 inches.

With the exception of School System E, where the difference between the two methods was approximately 20 miles, all difference are very small.

In regard to the total number of miles students were transported, the heuristic model gave better solutions. This condition may result simply from the model requiring the designer to be conscious of minimizing the number of miles students travel rather than the distances the buses travel. In this study no attempt was made to take the routes designed by the experienced school personnel and to minimize them in student miles. At this time, one can only conclude that the heuristic model will yield a solution comparable with that devised by a sample of experienced school transportation officials.

Evaluation of Heuristic Model

The evaluation of the heuristic model will follow the same outline that was given above in the discussion of the criteria by which a school bus routing method may be evaluated.

The heuristic model has only two steps that are rigidly defined, one, locating the starting point of a route and two, computing the shortest route to the school. There are three steps that are rather ambiguous: identifying isolates, selecting a school bus to be used on a particular route if different size buses are permissible, and minimizing student miles after the pick up points have been selected.

By definition, an isolate is any child who lives near the shortest route and who normally would not be picked up on subsequent routes. An attempt was made to define an isolate in terms of adjacent distances or the number of road miles to the nearest child who has not already been assigned to a school bus. A child was considered an isolate if a particular route doubled an adjacent distance. In many cases this rule-of-thumb was satisfactory.

The definite order of assigning school buses according to size cannot be determined at this time. Normally school buses of the smallest capacity were

assigned first and the largest buses last. This was done in the case of the heuristic solution to the six school systems. On the other hand in the miniature school systems, more efficient routes could be devised by not pre-determining the order of arranging buses. This item will be further discussed under assumptions.

The process of minimizing student miles after the pick up points had been selected has not been rigidly defined. Generally this factor was only considered in selecting the actual starting point of a route. The route was scanned for the three or four pick up points that were the greatest distance from the school. Then the number of children at each of these three or four points was noted; and if all points were approximately the same distance from the school, the point with the fewest children was the starting point. This was followed by the point with the next fewest number of children. In most cases the points under consideration were in the periphery of the school district.

The heuristic model as described herein requires one to assume that the child residing farthest from school should be given consideration over every other child in the school district. (Although this seems to be a reasonable assumption, it is believed that a method that requires assumptions to be made violates scientific procedures.) Therefore, a small bus should be assigned first since a student living the greatest distance and assigned to a small bus would be more likely to spend less time in route to school than if he were assigned to a large bus.

Although no evidence is submitted to support such a statement, it is believed that the heuristic procedure is readily understood. Clerical assistants can be taught to design satisfactory route systems.

The heuristic algorithm solutions can be easily computed. If one is given a transportation map with approximately 1000 children, a solution can be generated within five or six hours.

Since geographic features are not included in the model, it is not affected by such factors.

One favorable feature of the heuristic solution is that it may be stopped at any step, put aside, and started at a later date without any review.

Since the heuristic model was designed to generate one solution, there is no proof that such a solution is either a near optimal or optimal solution. Evidence has been shown that routes devised by such a method are slightly longer in terms of bus miles than solutions suggested by experienced personnel. On the other hand, evidence has been submitted which suggests that the total number of student miles can be greatly decreased by the heuristic method.

The heuristic model utilizes a hand solution and can be accomplished without office equipment. However, the use of an adding machine would reduce the time spent in computing bus and student miles. Therefore, this method is inexpensive.

In summary, the heuristic method of devising school bus routes accomplished that for which it was intended: providing a routing system with which to compare other systems. There is some evidence that it will give better routes than experienced personnel can produce; and since it is somewhat mechanical in nature, the laborious task of designing school bus routes can be delegated to clerical assistants. Furthermore, it can be used to generate several different routing systems by systematically varying the order by which buses are assigned. This increases the probability of securing a near optional or optional route.

Chapter 9. Combinatorial Model

The combinatorial model will be described first in a simplified narrative form, second by illustrating its application to the miniature school system problem and third by a flow chart.

Throughout these descriptions certain names, letters and symbols will appear within parentheses. These items will be used in the flow chart and are given here in order to acquaint the reader with their use and meaning.

Description of Combinatorial Model

The combinatorial procedure may be described as visualizing a group of bus drivers (K) sitting in a room each with a clip board containing several sheets of paper. Each sheet of paper is lined. Each line is numbered (LEVEL) and will be referred to as "slot." In front of the drivers is a large black board (LIST) on which appears each pickup point (I) and the number of students (KIDS) at each point. Each bus driver (K) with a clip board (LEVEL) represents a particular school bus (K). The director of this meeting (PROGRAM) begins by calling off the first pickup point (I) on the black board (LIST) and this point (I) is assigned to some driver representing bus (K). The bus driver (K) writes the name of the pickup point (I) on the first line of his first sheet on his clip board (LEVEL) to indicate that he now has one (LEVEL ONE) pickup point (I) on his bus (K). The driver (K) now checks to see if the number of students (KIDS) at this pickup point (I) will be enough to fill (LEAST) his bus (K) or if the number of students exceeds the maximum (MOST) of his bus (K). When the children (KIDS) are put on a bus (K) and added to the ones already on the bus (K), the sum is called (LOAD). When the driver (K) agrees

that he can transport the children (KIDS) at the pickup point (I), the director marks off the pickup point (I) on the black board to indicate the pickup points not assigned (LEFT). If the driver (K) states that he cannot transport the children (KIDS) at this pickup point (I), the director simply goes on to the next pickup point (I).

The director now reads another pickup point (I) and asks if the same driver (K) can transport these children (KIDS). If the bus driver (K) can, the children (KIDS) at the pickup point (I) are assigned to the bus (LOAD) and the pickup point (I) is marked off the black board (LEFT).

This procedure is continued until one bus is filled. Then the assignment to a second bus is made and all children are assigned to a particular bus. At this time all drivers compute the minimum number of miles they will have to drive to complete the bus route and the minimum number of student miles on the route. These figures (COST) are then recorded (STORED) on the sheet of paper on the clip board and collected for future comparisons with subsequent designed bus routes. It is to be noted that this routine assumes a full distance matrix has been made; that is, distances between all points have been computed. This item will be discussed later.

After the sheets of paper have been collected from each driver, the names of all pickup points and number of children at each point are again recorded on the board and re-assigned in such a manner that different combinations are used. This whole procedure is then repeated again and again until all feasible bus routes have been tried and respective bus and student miles have been recorded.

Illustration of Combinatorial Model

The miniature school system was altered in order to provide a condition that would likely be encountered in normal situations, i.e. a large number of children at one pickup point necessitating the assignment of two buses to that

particular point. The alteration involved the partitioning of the 12 students at point #4. This point was selected because it was the largest pickup point in the school system. No systematic procedure is known as to how a number of students should be partitioned and this subject will be discussed later. For this study the 12 students at point #4 were divided into three groups of 1, 2 and 9 students each. The division of students at point #4 produced a new configuration of 10 points, which are given in Figure 2.

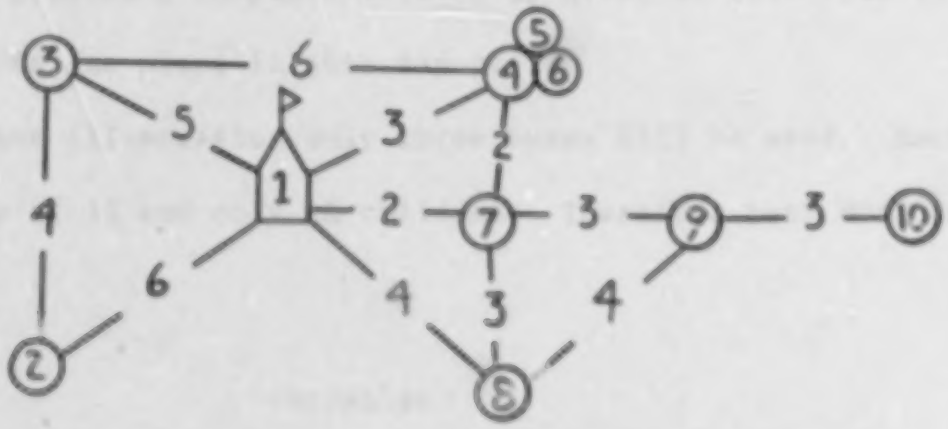


Figure 2. Skematic Diagram of Miniature School System with Point No. 4 Partitioned

From the new configuration the following distance matrix may be written:

NAME (I)	KIDS (I)	MILES
1	-	
2	6	6
3	4	5 4
4	1	3 9 6
5	2	3 9 6
6	9	3 9 6
7	10	2 8 7 2 2 2
8	9	4 10 9 5 5 5 3
9	3	3 4
10	1	3
		1 2 3 4 5 6 7 8 9 10

Not all distances are included in the matrix. An inspection of the miniature system indicates that there is little probability that a bus route would

go from point #2 to #10 without any intervening stops. Therefore, as a person prepares a matrix, it is believed that the only distances that need to be included are those that would likely be used. However, this aspect needs investigation.

Given below is a detailed description of the application of the combinatorial model to the miniature school system problem.

Although not all the codes will be used in the combinatorial solution of the immediate problem a complete listing is given at this time in order that they appear in one place in this report.

In the present illustration only three buses will be used. Each bus will have the capacity of 15 and only 15 children. Therefore both MOST and LEAST will be 15.

Variables

Names	Remarks
NAME (I)	Name of pickup points. In the miniature system: 1 2 3 . . . 10
K	Name of school bus. In the miniature system three and only three buses are to be used: Buses 1, 2, and 3.
LOAD (K)	Number of students that have been assigned to a bus.
LEAST (K)	Least number of students (LOAD) that may be assigned to a bus (K).
MOST (K)	Most number of students (LOAD) that may be assigned to bus (K).
KIDS (I)	Number of students at pickup point (I).
ILEFT	Name of pickup points (I) that have not been assigned.
KLEFT	Name of buses (K) that have not been assigned.
LEVEL (L, K)	The nth pickup point assigned to a particular bus.
LFIX (K)	Pre-assigned pickup point on bus (K). The school will be pre-assigned to all buses. Also in order to generate different routes, pickup points will have to be pre-assigned

	on subsequent routes in different combinations than on past routes. (The Monte Carlo method is one approach to pre-assigning points which will be discussed later.)
KKOST	The minimum number of bus miles and the minimum number of student miles required to transport the students to the school for bus K.
KOSTB	Best total cost or score to date, i.e. the minimum of minima for the KOST (K).
LLMIN	Current adjusted minimum load that may be assigned to current bus.
LLMAX	Current adjusted maximum load that may be assigned to current bus.
ILAST	Name of last pickup point. This point has the largest numerical identification number.
IBEG	Beginning pickup point I on bus (K).
ROUTE	Name of pickup points on current route.
LROUTB	Best current route to date.

Sequentially, the algorithm will consider the first bus by setting K = 1 and will attempt to place pickup points into its slots or LEVELS until a load of 15 children is reached. It will not stop with any lesser load; and if the addition of a particular pickup to its list would exceed the allowable load (15 students in this case), the algorithm will not add it but will attempt to consider the next point (i.e. I + 1). Should the program continue until all points have been considered I = LAST + 1, the algorithm returns to the last pickup point assigned, removes it from the bus, decreases the load by the number of children at that point, and makes appropriate changes in the LEFT matrix. At this point the program continues until a load is obtained or until all the combinations have been tried.

Bus #1 or K = 1 is now assigned to a route; and since the school is assigned to each bus, I is initialized to pickup point #2 and now an attempt is made to

assign a point into the first slot or LEVEL = 1 on K = 1. It has been convenient to write this information as LEVEL (1,1), denoting bus #1 and slot #1. As #2's six children do not exceed bus #1's capacity, I = 2 is placed in LEVEL (1,1) and a one is placed in LEFT (2) to indicate that point #2 is on a route. LEVEL is now increased to (1,2) and I is again initialized to point #2. The LEFT column is scanned and LEFT (2) indicates this point is already assigned and I is increased to #3. (A zero in LEFT indicates I has not been assigned and a one indicates I has been assigned.) Point #3's four children added to the six children already in LOAD, increases the load to 10, which is satisfactory. At this point I = 3 is placed in LEVEL (1,2) and appropriate changes are made in LEFT (3). LEVEL is now increased to (1,3). I begins its search again from I = 2, and stops at I = 4. Point #4 has one pupil and after the various operations, 4 is added in LEVEL (1,3). I = 5 is likewise added in LEVEL (1,4).

An attempt to add I = 6 with its nine children exceeds the capacity of bus #1, as does I = 7, 8, 9. Now I = 10 can be added to LEVEL (1,5) for a total of 14 children. I is now incremented to 10 + 1 or past its maximum or beyond LAST, and an examination of its load indicates an unacceptable combination of pickup points because LOAD = 14 and LEAST = 15.

The last acceptable point that was assigned is found in LEVEL (1,5), or pickup point I = 10. This point is now removed and LEVEL is reduced by one to LEVEL (1,4). I is incremented by one to I = 10 + 1 (an overdraft) and a test of the current load of 13 indicates it is less than LEAST or too small. The point in LEVEL (1,4) is now removed from the list (pickup point I = 5) and LOAD is changed accordingly to a total of 11 students. I is incremented to #6; and when the nine children are added to LOAD, an overdraft (of 20) results, as would also be produced by #7 and #8. It is found that I = 9 can be added giving LOAD = 14 and thus #9 can be placed in LEVEL (1,4). I = 10 can be added to LEVEL (1,5) for a total load of 15 children. This is the first acceptable

solution for the first bus. Control is now given to a subroutine to determine the best routing for this list of pickup points according to the criterion of "best" student miles. For student miles, the best routing would be 10-9-4-3-2-1 for a total score (KKOST) of 179 student miles.

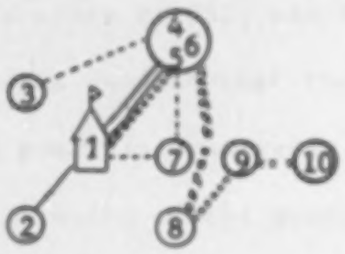
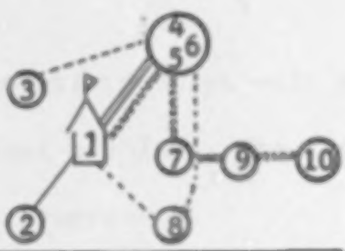
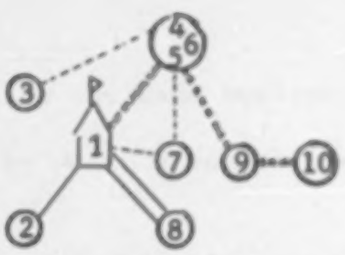
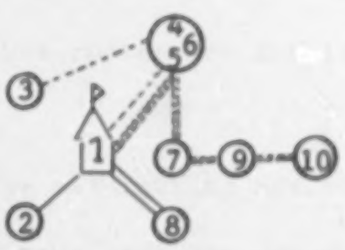
K is increased to 2 to consider the points for the second bus and LEVEL is set to read (2,1). The LEFT (I) column indicates that I = 2, 3, 4, 9, and 10 have been assigned. It is found that I = 5 could be added to LEVEL (2,1), and likewise I = 6 can be added to LEVEL (2,2) giving a current load of 11 children. Further combinations all prove unsatisfactory; thereby indicating that the current combination assigned to bus #1, or the first bus, is also unacceptable. Therefore, K is decreased to 1. I = LEVEL (1,5) = 10 is now removed from the list and this whole process is repeated until a satisfactory solution is determined for the first bus. The first acceptable solution is LEVEL (1,1) = 2 and LEVEL (1,2) = 6. The score or cost for this route is 99 student miles.

Once again K is incremented to 2 and by the same process an acceptable combination of points is found whose students total 15. The points 3, 4 and 7 are assigned to bus #2 with a total of 64 student miles. The remaining points as given by LEFT (I) total 15 children. The student miles are computed to be 129; thus giving a route system 292 student miles. Table 2 gives the values placed in the various Levels as well as a graphic representation of this solution.

To generate a second acceptable routing system, K is reduced to 2 and by the use of pre-assigning points based on the first solution, different combinations are produced. Subsequent solutions that can be generated are also given in Table 2.

When the program generates a route system whose student miles exceed the "best" (KOSTB) system to date, the alogrithm replaces the previous solution "best" with the present solution and begins generating another route; therefore,

Table 2. Illustrations of Combinatorial Algorithm in Solving Miniature School System Problem

Solution	Level	Bus #1	Bus #2	Bus #3	Miles Student	Miles Bus	Map
#1	1	2	3	5			
	2	6	4	8			
	3		7	9			
	4			10	292	37	
#2	1	2	3	4			
	2	6	5	7			
	3		8	9			
	4			10	301	38	
#3	1	2	3	5			
	2	8	4	6			
	3		7	9			
	4			10	252	35	
#4	1	2	3	4			
	2	8	5	7			
	3		6	9			
	4			10	265	37	

only the best solution to date is stored. The procedure stops when all possible combinations have been tried and K is reduced to zero.

The apparent advantage of the combinatorial over the other methods is rather obvious when one compares the "student miles" of the three procedures of devising school bus routes. It is to be recalled the best solution among graduate students was 261; the heuristic model yielded a score of 255; and the best score to date by the combinatorial is 252. These data suggest that the "best" solution can only be obtained by a combinatorial analysis. An even "better" solution might be achieved by a different partitioning of the students at the various pickup points. Only one point (#4) was partitioned in the miniature system.

At the present time a computer program has not been written that will give a completely satisfactory solution to the miniature school problem. The flow chart given in Figure 3 gives the present status of the program.

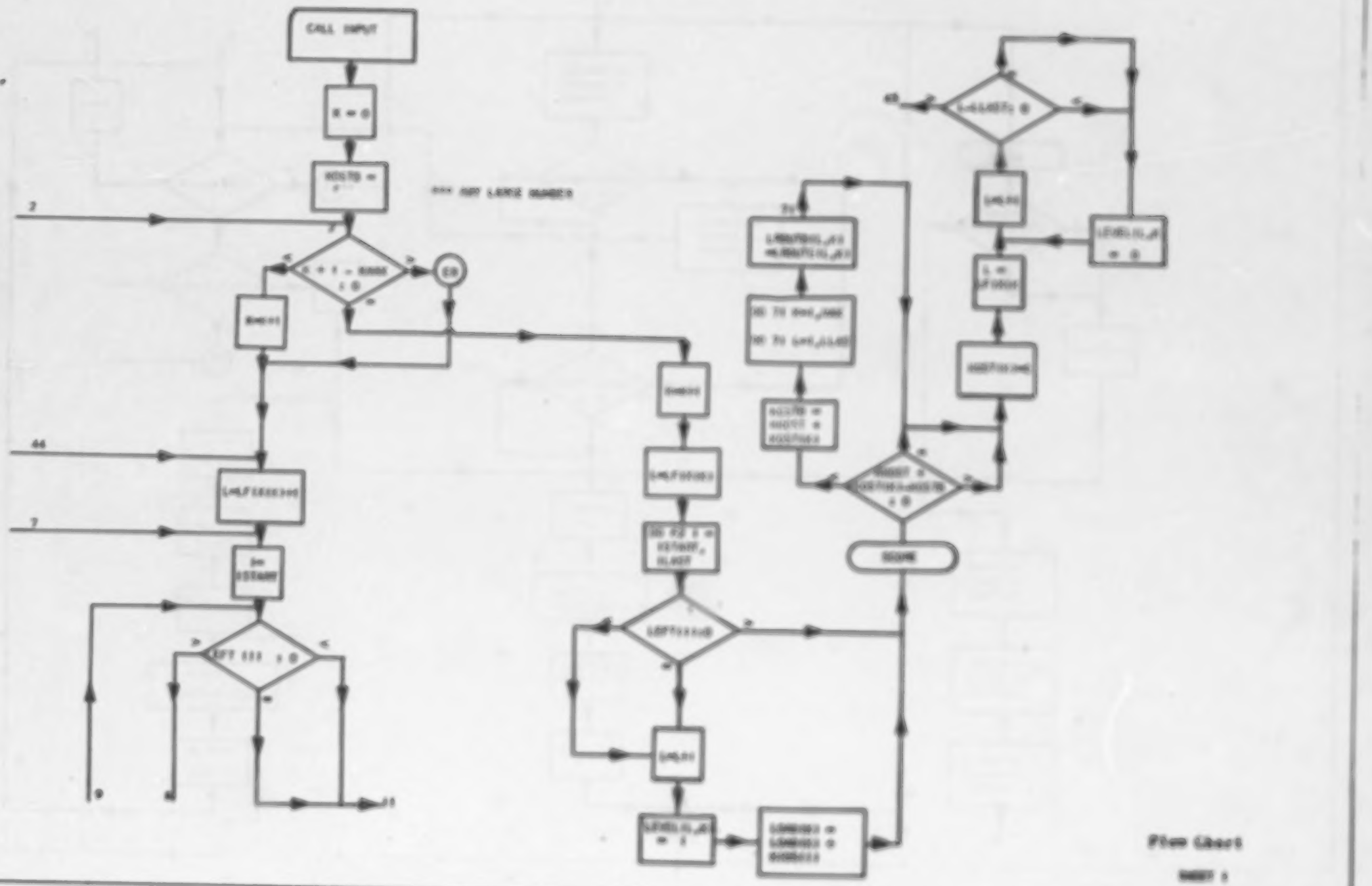
Evaluation of Combinatorial Model

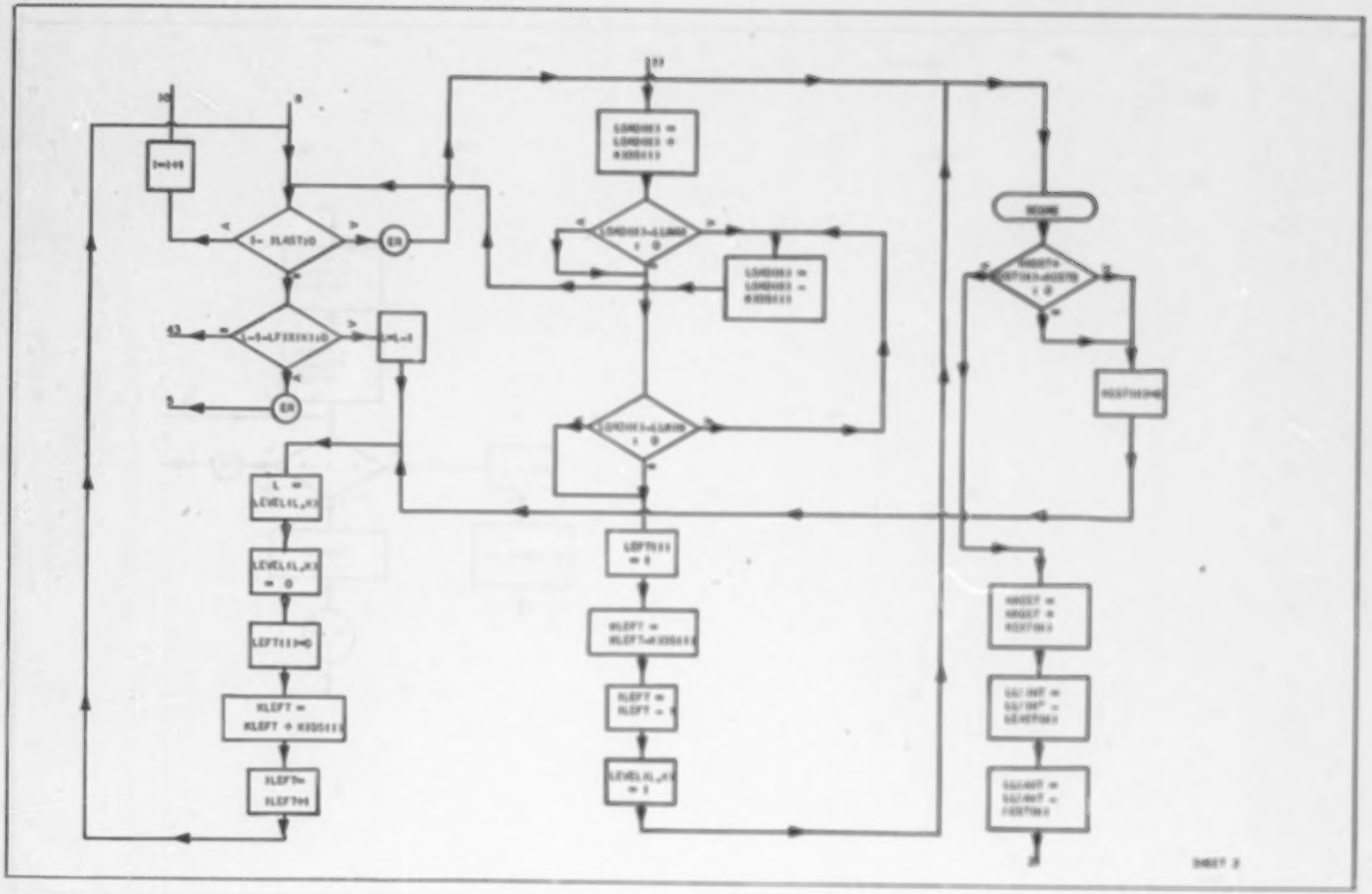
The evaluation of the combinatorial model will follow the same outline that was given above in the discussion of the criteria by which a school bus routing method may be evaluated.

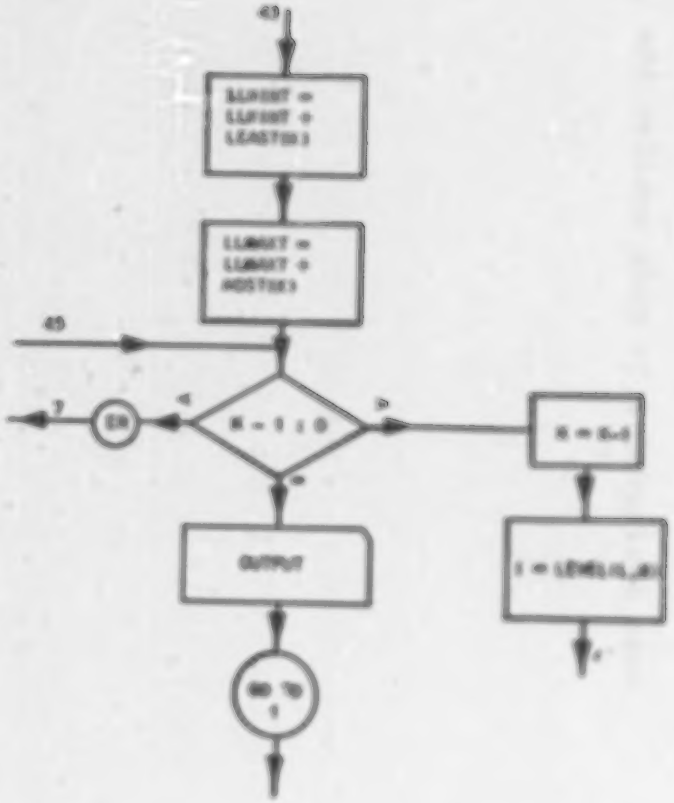
The combinatorial algorithm is rigidly defined, as is any computer alogrithm. To date the only other know methods of devising school bus routes are intuitive in nature.

The combinatorial method makes no assumption because all routing systems are generated. From a theoretical standpoint, there may have been one assumption made in this study, that is, an optimal school bus routing system does exist for a specific criterion.

The combinatorial model is not readily understood because it is highly complex. Also, this model is the first attempt to program school bus routes. On the other hand, the results are easily understood.


CALL INPUT
*** NOT LEGAL NUMBER
Flow Chart
SHEET 1





SHEET 3

The combinatorial model should be easily computed. Although the explanations may be long and complicated, almost all the computations involve decrementing which is an exceedingly fast operation. The philosophy is that sufficient plausible restrictions exist and can be introduced to keep running time within economic bounds.

Differences in road patterns and terrain will have no effect on the solution provided by the combinatorial method because all road patterns are considered, not one particular type.

The criterion relating to starting and stopping the routing of a bus does not apply to methods developed for high speed electronic computers.

In the generation of all solutions, it can be assumed that if an optimal solution does exist, it will have been generated and can be so identified.

The cost of devising school bus routes by the combinatorial model cannot be determined at this time. It is believed that by experimentation with different high speed electronic computers, and in the refinement of the present program, this method of determining routing systems will enable schools to save considerable money, and relieve them of a laborious task.

Chapter 10. Summary and Recommendations

The purpose of this investigation was to develop a set of sequential steps, or a program, that would devise optimal routes for school buses.

The evaluative criteria for a transportation system and for a method of devising school bus routes were suggested.

One of the two models developed for this study, called the heuristic model, was a hand procedure and based on the assumption that in an efficient school bus system, student miles should be minimized rather than bus miles. The other model was designed to be used with high speed computers and is called the combinatorial algorithm. The latter model also minimized student miles.

Given a map of a typical school district, six experienced school officers were asked to design school bus routes according to their usual practice. Also, a class of graduate students designed routes for a miniature school district. The results of these routes were compared with routes designed by models developed for this study.

The heuristic model yielded routes whose bus miles were comparable with those routes designed by experienced designers. A noticeably smaller number of student miles was obtained by the use of the heuristic model. Therefore, it may offer a rapid means of obtaining a reasonably good routing system. Since it is mechanical in nature, clerical assistants may be trained to design tentative routes.

The combinatorial algorithm was described and illustrated. It was demonstrated that this model provided a better solution than either the heuristic model or graduate student designers. The flow chart for the combinatorial

model was given. At the present time a completely satisfactory program has not been written for a high speed electronic computer.

The three major results of this project are:

1. The routing of school buses can be reduced to a set of sequential steps that probably can be programmed for modern electronic data processing machines.

2. The models developed in this project can generate different routing systems using different sizes of buses. By comparing the bus miles and student miles for each routing system, the most efficient combination of buses can be identified. The combinatorial algorithm was designed to generate all possible routes, which would include the optimal routing system.

3. The models suggested in this study utilize variables that are applicable to all school systems.

Some of the more general recommendations suggested by this project are as follows:

1. The flow chart in this report gives a program that compares each solution with the best solution to date and punches out only the best solution. A feature that should be investigated is a subroutine that would select 10 near-optimal solutions instead of one solution. This will present to the transportation officer a list of good routes.

2. A simplified subroutine is needed to select the order of pickup points that would minimize bus and student miles. At present, Riley's (1959) suggestion seems to be the most promising method that might be adapted for this purpose.

3. The number of solutions for the school bus problem that can be generated is unknown at this time. In fact, no known means of estimating the number of different routes exists. Since even the theoretical size of the population is unknown, it is also unknown how many routes need to be generated to secure

sufficient data upon which near optimal routes could be extrapolated. Therefore, more information is needed concerning the parameters of the solution.

4. In the present study the number of pickup points was limited to approximately 100 points per school system. It was found that the number of children at certain points had to be partitioned in order to fill school buses. No problem arose in the heuristic solutions. In the combinatorial algorithm a method is needed to partition these children either concurrently with the solution of the problem or at the input stage. Some experimentation was done in partitioning the number by an addend method, i.e. 17 would be partitioned into 1, 2, 2, 4, 8. This method greatly increased the number of pickup points and thereby increased the computer time. It is believed that the concurrent method would be a more satisfactory procedure and warrants extensive investigation.

5. The program suggested in this report first generates school bus routes by considering only the number of children (LOAD) and then computes bus and student miles for evaluative purposes. An attempt should be made to determine the feasibility of a program that would consider number of students and distances concurrently.

6. The criteria were discussed by which a school bus route system could be evaluated. Only bus and student miles were used to appraise the routes. Although it was inferred that routes should be evaluated in terms of student variables rather than equipment and financial factors, the concept of student miles may not be the best criterion. Before any extensive research is undertaken, these criteria themselves should be evaluated.

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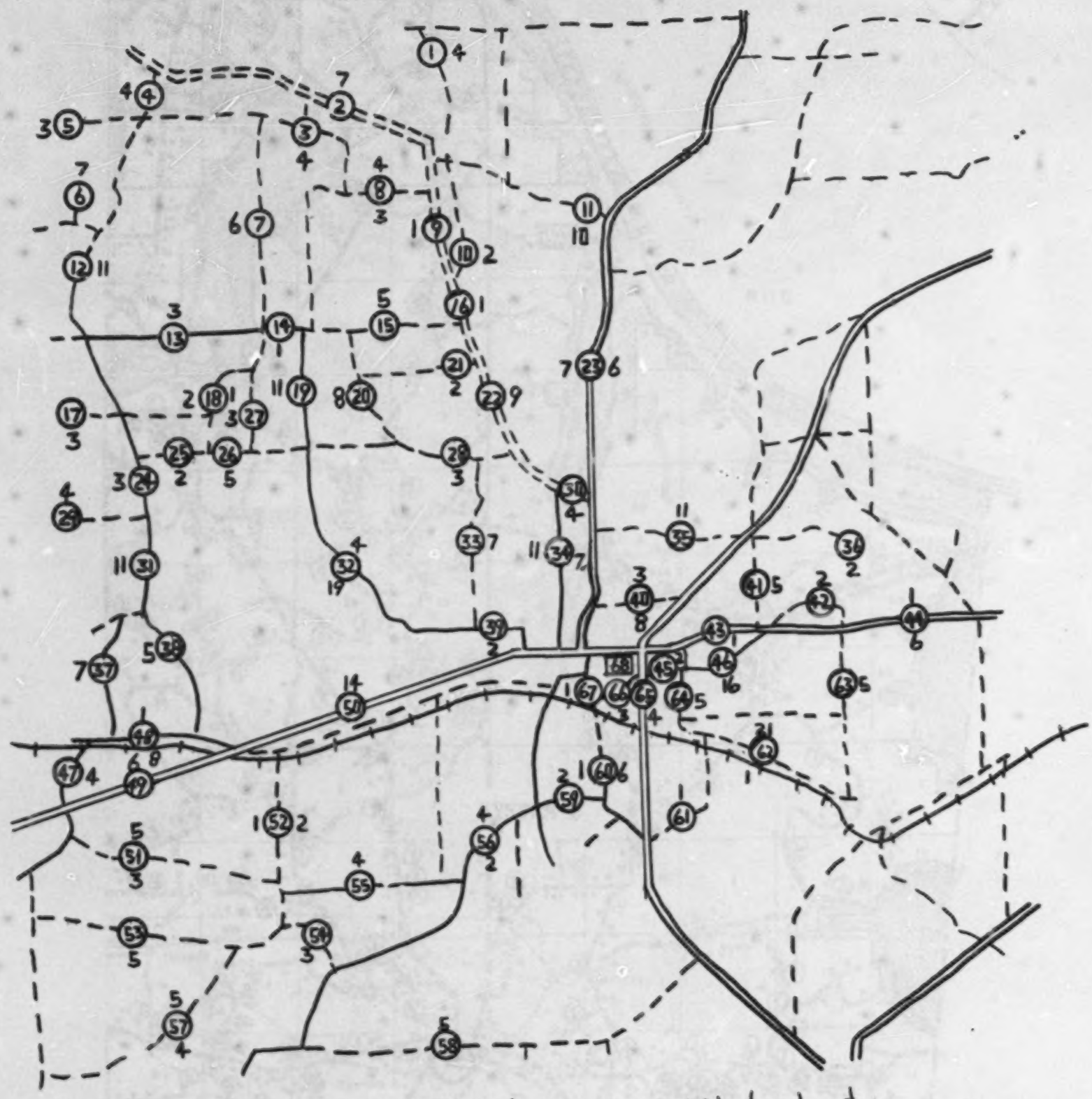
2

Appendix A

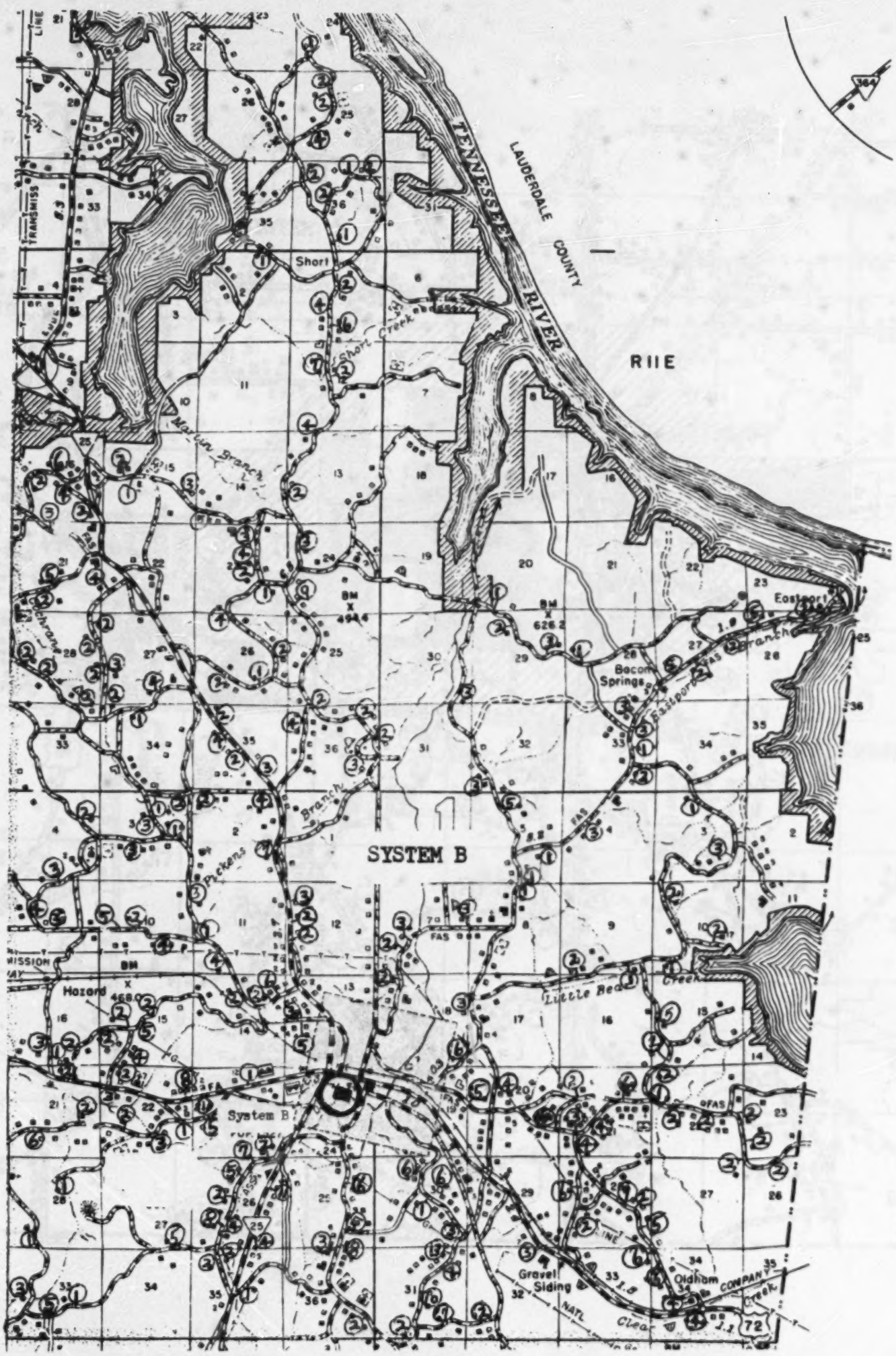
Maps of the Six School Systems

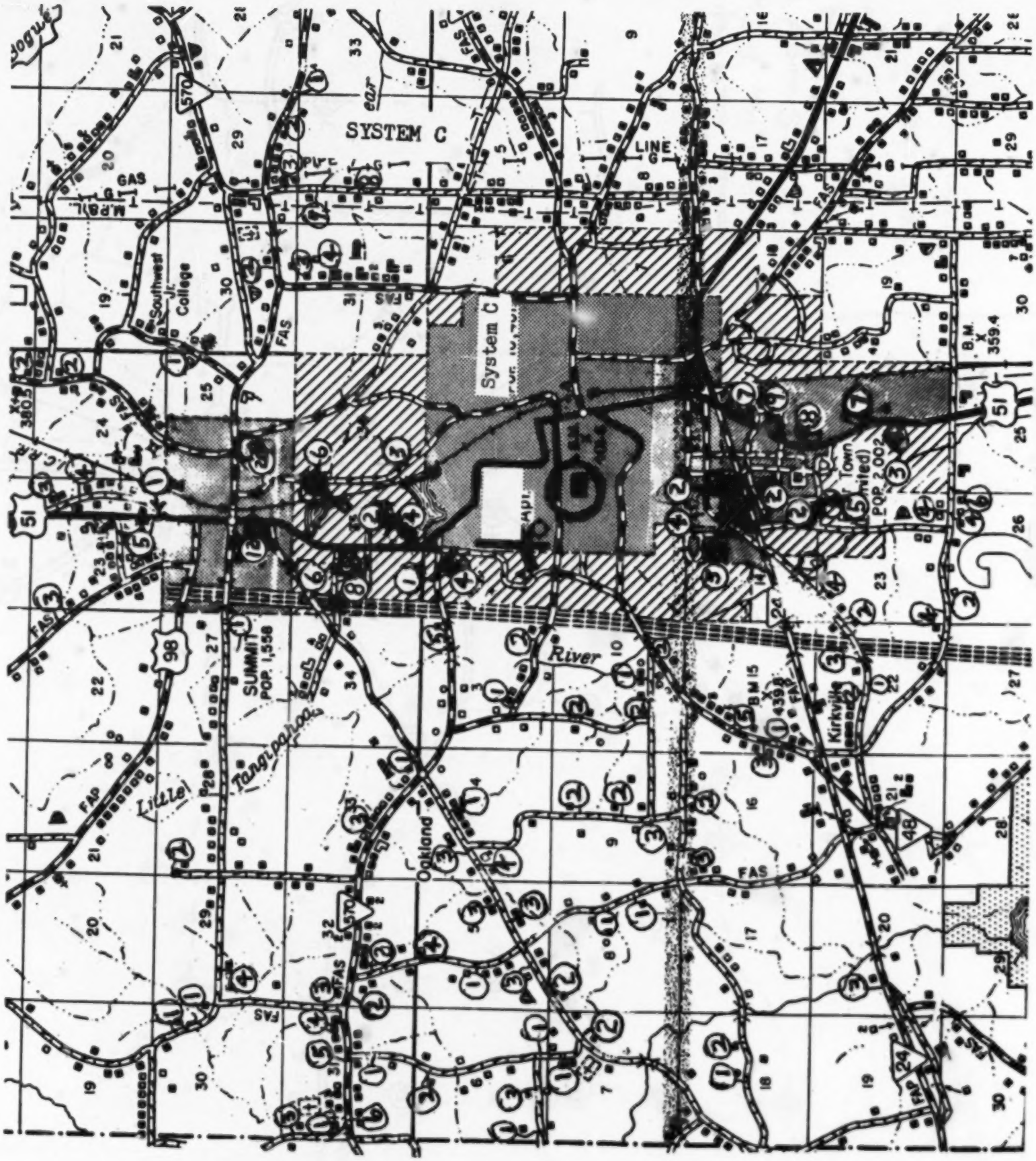
SYSTEM A
BOLIVAR
WASHINGTON COUNTY

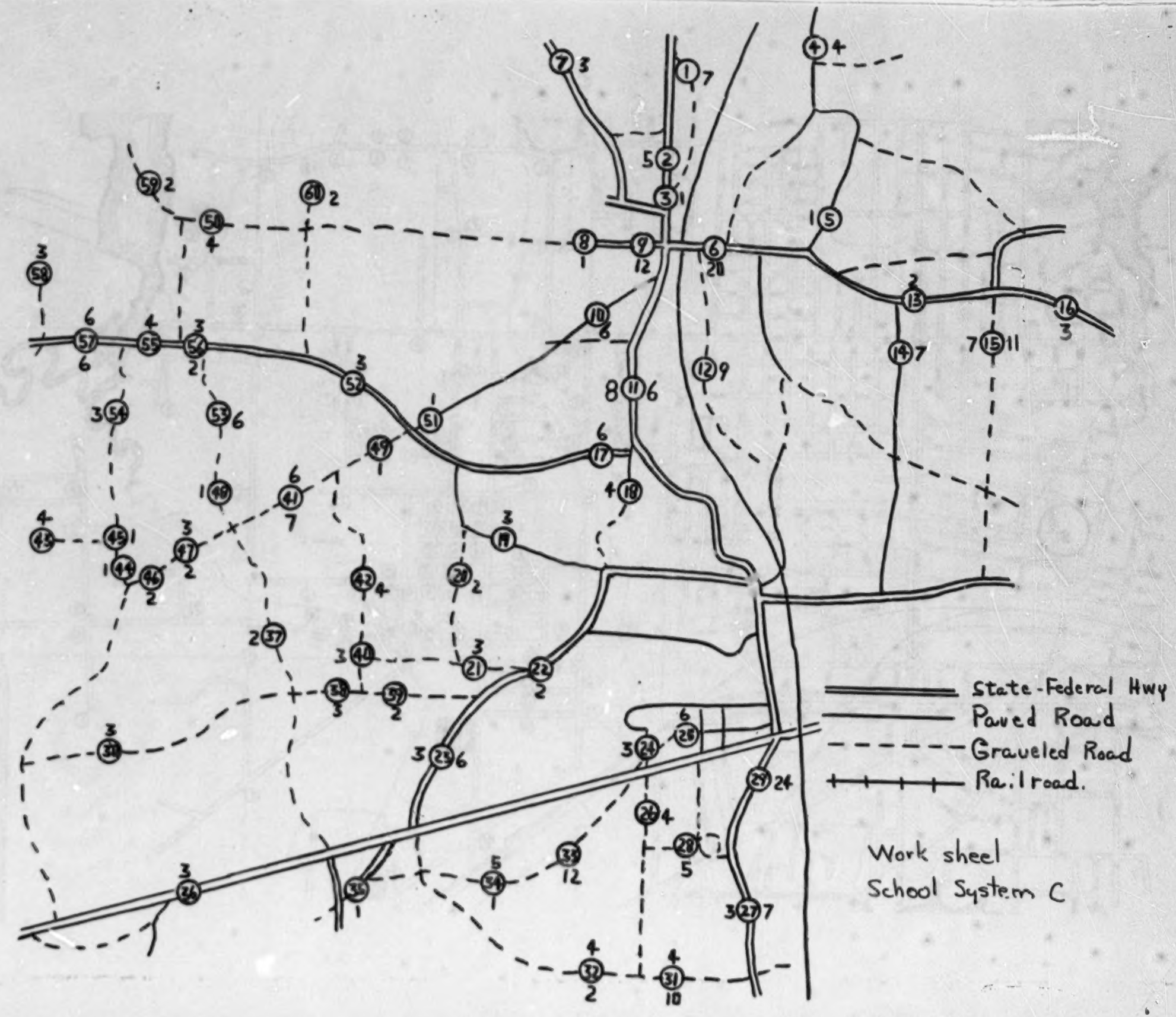


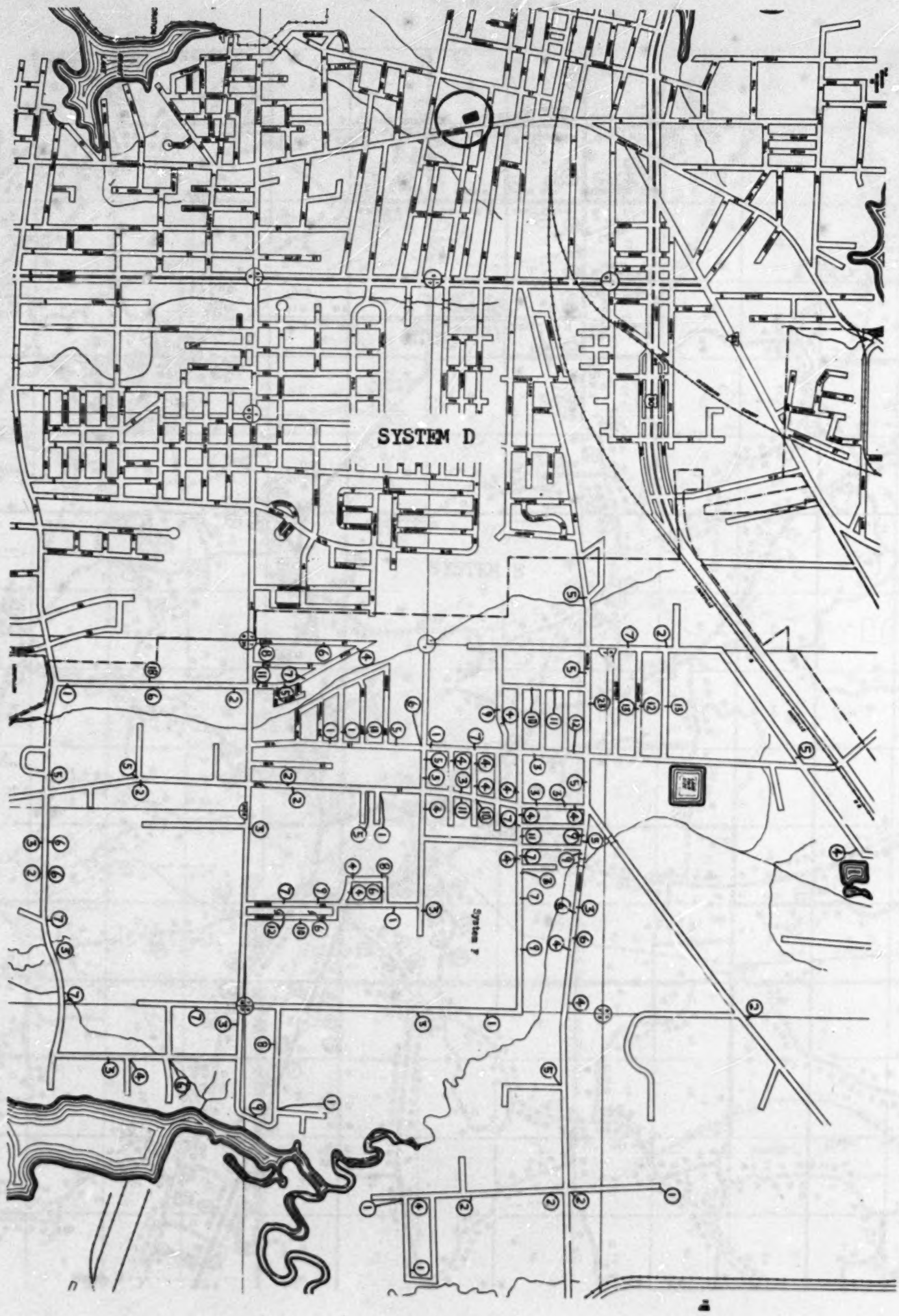
State and Federal Highway
Paved roads
Graveled roads
Railroad

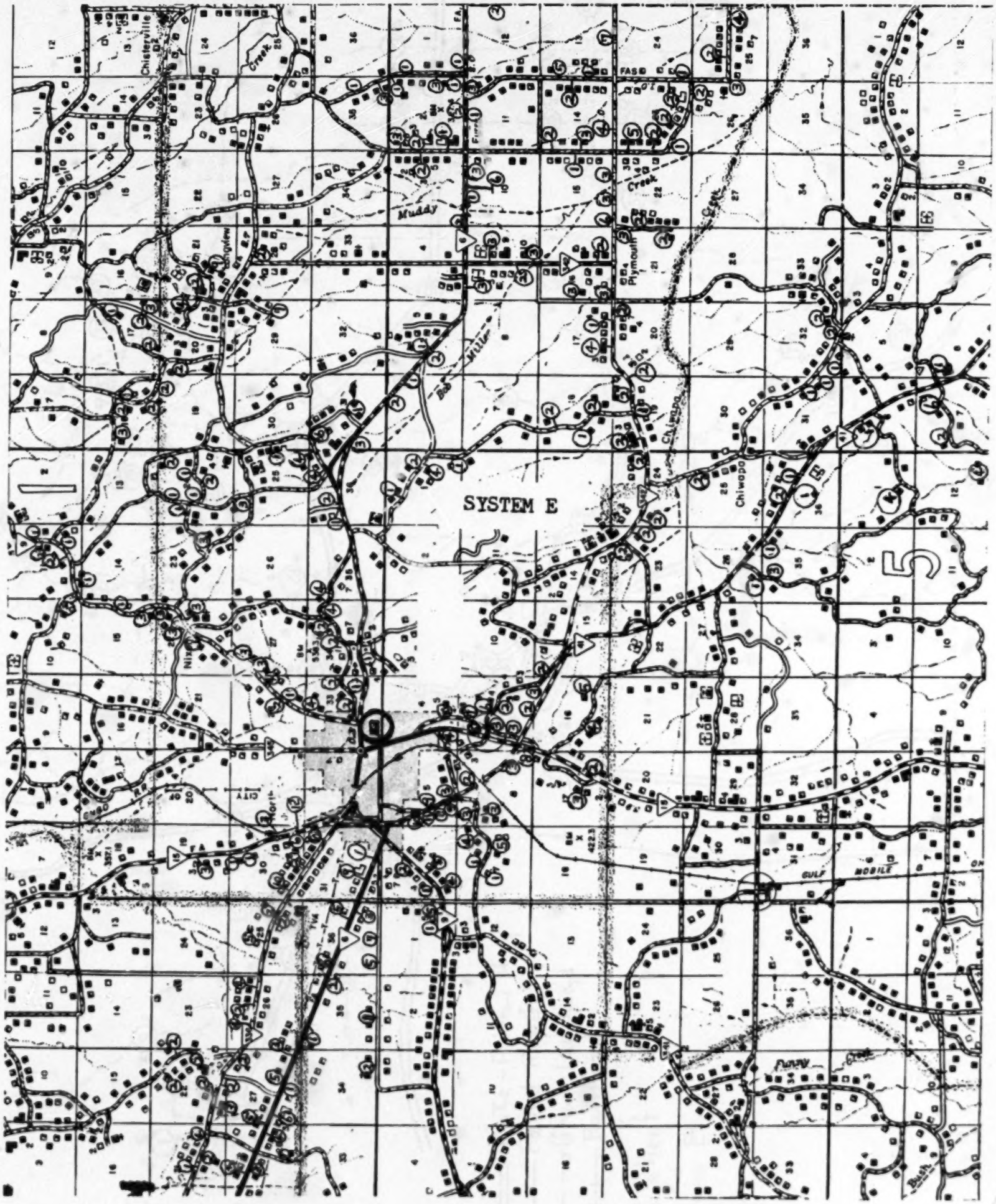
Work sheet
School System A

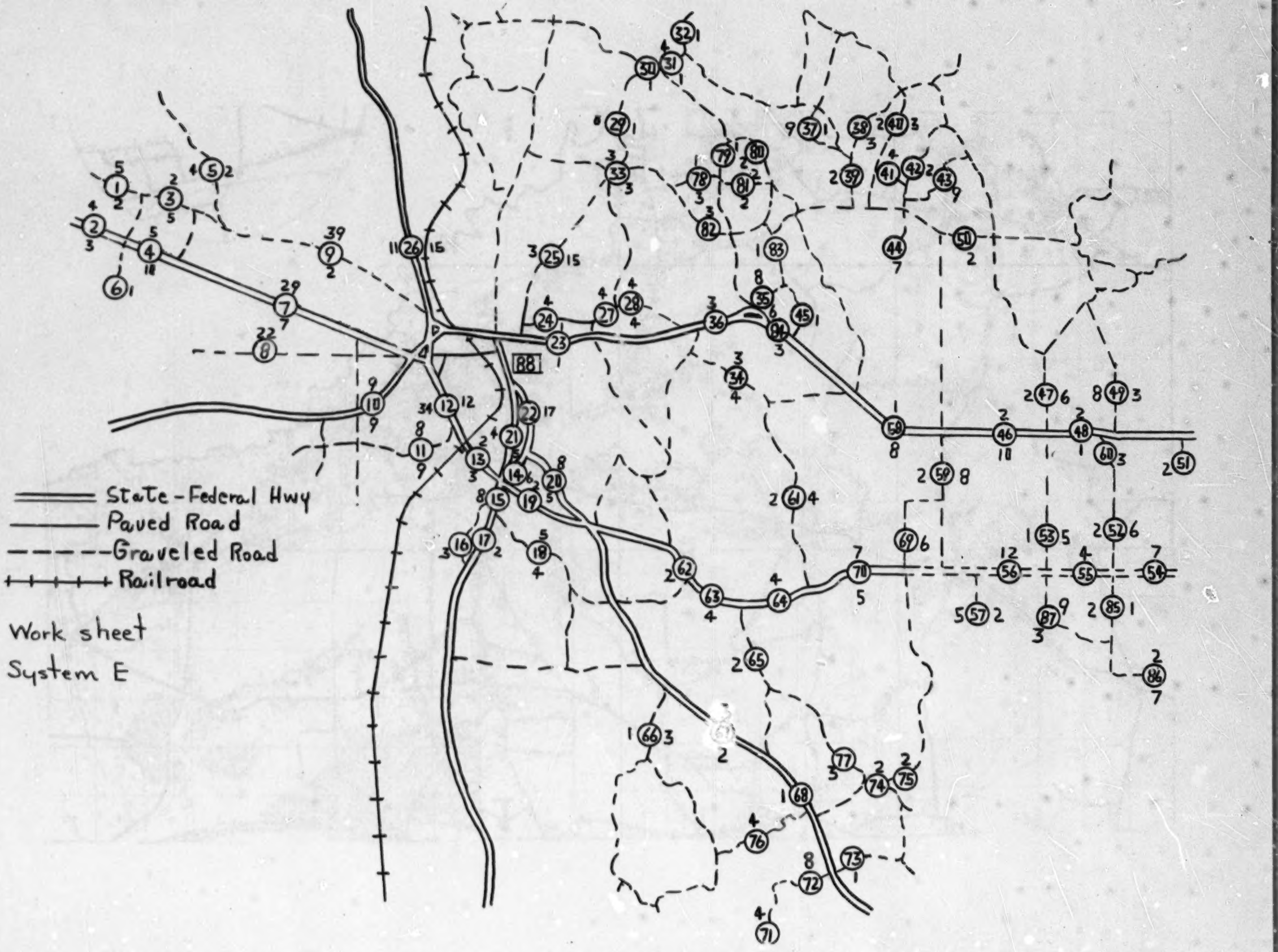

TENNESSEE RIVER
LAUDERDALE COUNTY
RIIE
SYSTEM B
Short
Short Creek
Martin Branch
Eastport
Bacon Springs
Little Bear Creek
Hazard
System B
Gravel Siding
Oldham
Clear Creek
NATL
COMPANY
FAS
TRANSMISS LINE

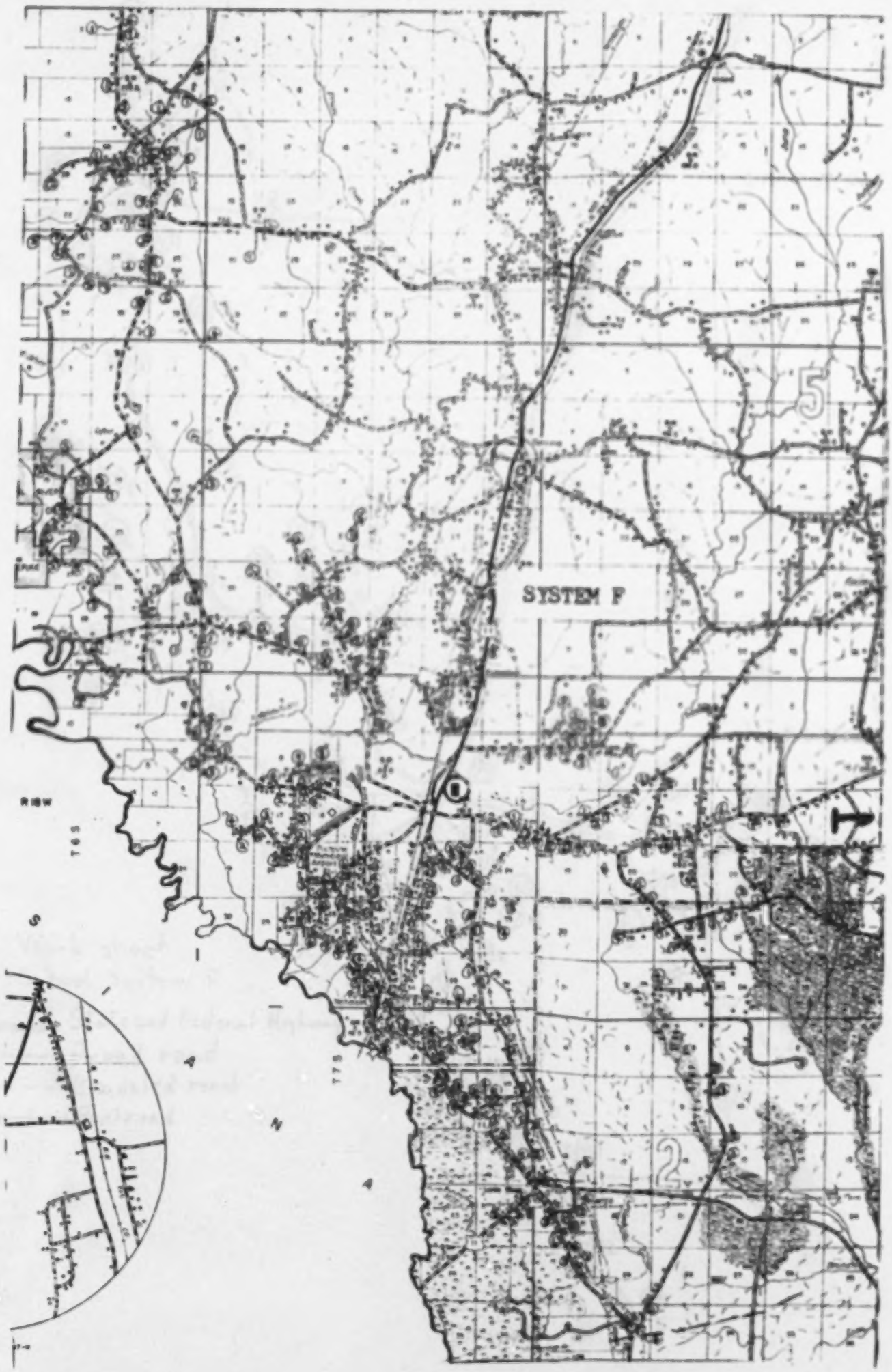

SYSTEM C
LINE
System C
Southwest Jr. College
SUMMIT
POP. 1,558
Town
POP. 2,002
Tangipahoa
River
Little
Oakland
Kirkville
FAS
FAP
FAS
51
98
570
B.M.
X
359.4

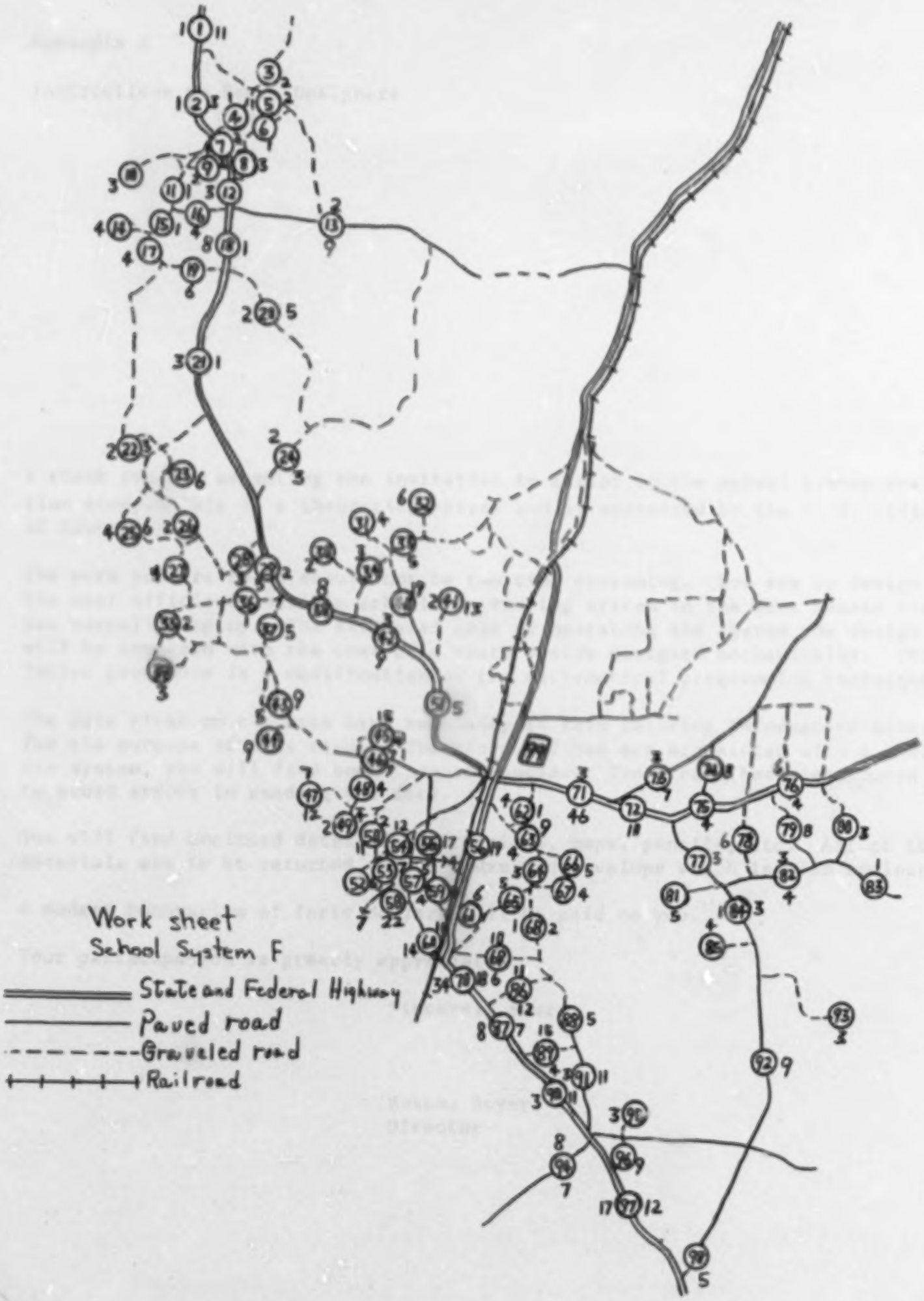

State-Federal Hwy
Paved Road
Graveled Road
Railroad.
Work sheel
School System C


SYSTEM D
System 1


SYSTEM E
Muddy
Creek
Chesterville
Plymouth
Chiwapa
Bee
Miller
GULF
MOBILE
Funny
5


State - Federal Hwy
Paved Road
Graveled Road
Railroad
Work sheet
System E
88


SYSTEM F
5
2
1
T
R8W
T6S
T7S
S
N


Work sheet
School System F
State and Federal Highway
Paved road
Graveled road
Railroad

Appendix B

Instructions to Route Designers

I thank you for accepting the invitation to assist in the school transportation study. This is a theoretical study and is sponsored by the U. S. Office of Education.

The work you are to do should not be too time consuming. You are to design the most efficient possible school bus routing system in the same manner that you normally employ. The estimated cost of operating the system you design will be compared with the cost of a route system designed mechanically. This latter procedure is a modification of the mathematical programming technique.

The data given on the maps have been adapted from existing information solely for the purpose of this study. Therefore, if you are acquainted with a particular system, you will find some inconsistencies. These have been introduced to avoid errors in reading the data.

You will find enclosed detailed instructions, maps, pencils, etc. All of these materials are to be returned in the addressed envelope which is also enclosed.

A modest honorarium of forty dollars will be paid to you.

Your participation is greatly appreciated.

Sincerely yours,

Roscoe Boyer
Director

INSTRUCTIONS FOR DRAWING BUS ROUTES

The following items are inclosed:

Enclosure #1: A large map for you to complete
Enclosure #2: A sample map with completed bus routes
Enclosure #3: Table A
Enclosure #4: Table B
Package of colored pencils
Addressed envelope

I. Description of Enclosures:

a. Enclosure #1 is a large map of the school district for which you are to construct routes. The circled red numbers recorded on the map indicate bus stops and the number of children to be picked up at each stop. A black encircled rectangle gives the location of the school to which all children are to be transported.

b. Enclosure #2 is an example of a completed route system for a school. After you have drawn routes on the large map (Enclosure #1) it will look similar to this sample (Enclosure #2).

c. Enclosure #3 is Table A, entitled "Time Spent on Assigning Bus Routes." Instructions are listed below the table.

d. Enclosure #4 is Table B, entitled "Route Information Table." This table will be used to evaluate the efficiency of manual and mathematical assignment of school bus routes. Instructions are listed below the table.

II. Instructions to be followed in drawing the routes on the large map:

a. You are to indicate the starting point of each bus route by writing in the small circle in the color of that particular route. This starting point may be at any location. You need not be concerned with how the bus gets to the starting point. See Enclosure #2 for an example.

b. You are to designate each route by recording an identification number at the starting point . This number will correspond to the data given in Table B. See Enclosure #2 for an example.

c. In cases where two or more buses pass the same pickup point, circle the red digit (denoting the number of children picked up at that point) with the color being used for the route of the bus they will ride. See insert on Enclosure #2 for an example.

d. You are to use only the following sizes of buses: 48, 54, 60, 66. You may use any combination of these sizes.

e. Your assignments are not to exceed the capacity of each individual bus.

f. You may use any type route: circular, shoestring, etc.

g. You may assume that any road or combination of roads may be used regardless of their construction or condition indicated on the map. Assume that all roads are equally good.

h. You are to assume that no child will walk and that all children will be picked up at points where the red line drawn from the encircled red number touches the road.

i. The total number of pupils on each map has been circled in red at the top of the large map (Enclosure #1). The total number of children assigned to buses in your route system should equal this number. You are to sum the number of children assigned to all buses to check if you have accounted for all children.

TABLE A

TIME SPENT ON ROUTING BUSES

Date	Time	Activity
(Example)		
December 10, 1960	12:30-2:00	Worked on routes 1, 2, 3

Instructions for Table A

1. The purpose of Table A is to record the time you have spent in route construction.
2. An example of the type of data desired has been recorded on line 1.
3. Do not include time spent in reading instructions and assembling materials.
4. Do not estimate your time. Please be exact.

TABLE B

ROUTE INFORMATION TABLE

Route number	Color	Bus capacity	Number of pupils assigned	Length of route one way
(Example)				